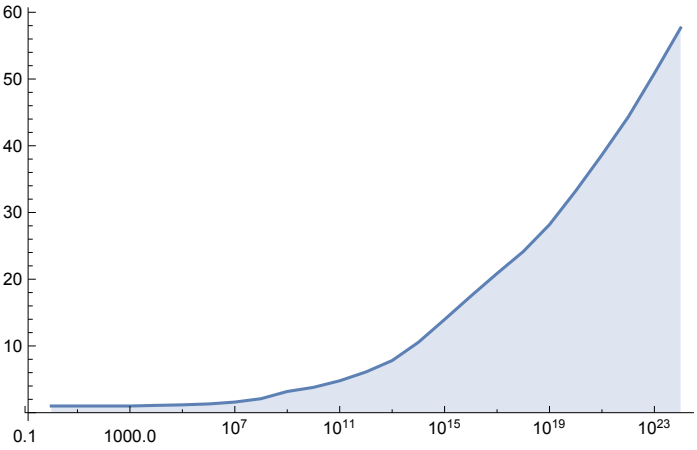
In[1]:=
```
(* List of fast Deleglise-Rivat alpha factors for x ≤ 10^24 found by
  running pi(x) benchmarks using the find_fastest_alpha.sh script *)

alphaDelegliseRivat = { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1},
  {10^3, 1}, {10^4, 1.095}, {10^5, 1.174}, {10^6, 1.310}, {10^7, 1.591},
  {10^8, 2.095}, {10^9, 3.178}, {10^10, 3.795}, {10^11, 4.779}, {10^12, 6.103},
  {10^13, 7.810}, {10^14, 10.545}, {10^15, 13.969}, {10^16, 17.461},
  {10^17, 20.862}, {10^18, 24.146}, {10^19, 28.184}, {10^20, 33.230},
  {10^21, 38.64}, {10^22, 44.296}, {10^23, 50.845}, {10^24, 57.647} }
```

Out[1]=
```
{{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1.095}, {100 000, 1.174},
 {1 000 000, 1.31}, {10 000 000, 1.591}, {100 000 000, 2.095},
 {1 000 000 000, 3.178}, {10 000 000 000, 3.795}, {100 000 000 000, 4.779},
 {1 000 000 000 000, 6.103}, {10 000 000 000 000, 7.81}, {100 000 000 000 000, 10.545},
 {1 000 000 000 000 000, 13.969}, {10 000 000 000 000 000, 17.461},
 {100 000 000 000 000 000, 20.862}, {1 000 000 000 000 000 000, 24.146},
 {10 000 000 000 000 000 000, 28.184}, {100 000 000 000 000 000 000, 33.23},
 {1 000 000 000 000 000 000 000, 38.64}, {10 000 000 000 000 000 000 000, 44.296},
 {100 000 000 000 000 000 000 000, 50.845}, {1 000 000 000 000 000 000 000 000, 57.647}}
```

In[2]:=
```
ListLogLinearPlot[alphaDelegliseRivat, Filling → Bottom, Joined → True]
```

Out[2]=



In[3]:=
```
(* alpha is a tuning factor that balances the compuation
 of the easy special leaves and the hard special leaves. The
 formula below is used in the file src/primecount.cpp to
 calculate a fast alpha factor for the computation of pi(x). *)

NonlinearModelFit[alphaDelegliseRivat,
 a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]
```

Out[3]=
```
FittedModel[ 1.3724 - 0.110407 Log[x] + 0.0018113 Log[x]^2 + 0.00033826 Log[x]^3 ]
```

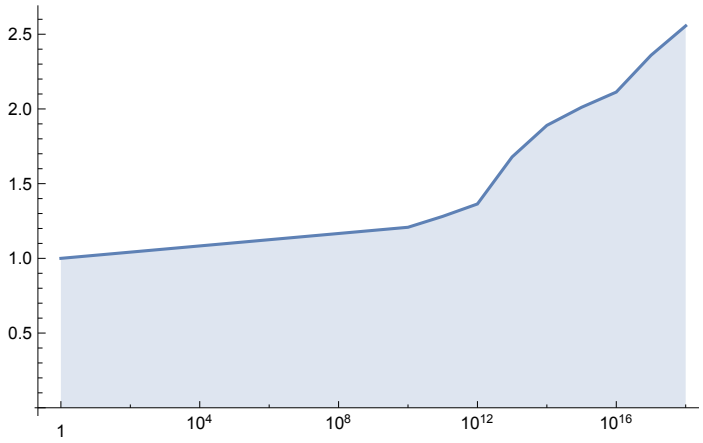
In[37]:=
```
(* List of fast Lagarias-Miller-
 Odlyzko alpha factors found by running pi(x) benchmarks. *)

alphaLMO = {(* {x, alpha} *) {1, 1}, {10^10, 1.208},
  {10^11, 1.281}, {10^12, 1.364}, {10^13, 1.679}, {10^14, 1.890},
  {10^15, 2.011}, {10^16, 2.113}, {10^17, 2.359}, {10^18, 2.556}}
```

Out[37]=
```
{{1, 1}, {10 000 000 000, 1.208}, {100 000 000 000, 1.281},
 {1 000 000 000 000, 1.364}, {10 000 000 000 000, 1.679}, {100 000 000 000 000, 1.89},
 {1 000 000 000 000 000, 2.011}, {10 000 000 000 000 000, 2.113},
 {100 000 000 000 000 000, 2.359}, {1 000 000 000 000 000 000, 2.556}}
```

In[38]:=
```
ListLogLinearPlot[alphaLMO, Filling → Bottom, Joined → True]
```

Out[38]=



In[39]:=
```
(* alpha is a tuning factor that balances the compuation
 of the easy special leaves and the hard special leaves. The
 formula below is used in the file src/primecount.cpp to
 calculate a fast alpha factor for the computation of pi(x). *)
```

In[40]:=
```
NonlinearModelFit[alphaLMO, a (Log[x])^2 + b Log[x] + c, {a, b, c}, x]
```

Out[40]=
```
FittedModel[ 0.990948 - 0.0261411 Log[x] + 0.00156512 Log[x]^2 ]
```